

# The quark-antiquark spectrum from upside down<sup>\*</sup>

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**Abstract.** We argue that the spectra of quark-antiquark systems should better be studied from higher radial excitations and, in particular, from configurations with well-defined quantum numbers, rather than from ground states and lower radial excitations, the most suitable system being charmonium.

In the Resonance-Spectrum Expansion (RSE) [1], which is based on the model of Ref. [2], the meson-meson scattering amplitude is given by an expression of the form (here restricted to the one-channel and one-delta-shell case)

$$\mathbf{T}(E) = \left\{ -2\lambda^2 \mu p j_\ell^2(pr_0) \sum_{n=0}^{\infty} \frac{|g_{nL(\ell)}|^2}{E - E_{nL(\ell)}} \right\} \mathbf{\Pi}(E) \quad , \quad (1)$$

where  $p$  is the center-of-mass (CM) linear momentum,  $E = E(p)$  is the total invariant two-meson mass,  $j_\ell$  and  $h_\ell^{(1)}$  are the spherical Bessel function and Hankel function of the first kind, respectively,  $\mu$  is the reduced two-meson mass, and  $r_0$  is a parameter with dimension  $\text{mass}^{-1}$ , which can be interpreted as the average string-breaking distance. The coupling constants  $g_{NL}$ , as well as the relation between  $\ell$  and  $L = L(\ell)$ , were determined in Ref. [3]. The overall coupling constant  $\lambda$ , which can be formulated in a flavor-independent manner, represents the probability of quark-pair creation. The dressed partial-wave RSE propagator for strong interactions is given by

$$\mathbf{\Pi}_\ell(E) = \left\{ 1 - 2i\lambda^2 \mu p j_\ell(pr_0) h_\ell^{(1)}(pr_0) \sum_{n=0}^{\infty} \frac{|g_{NL}|^2}{E - E_{NL}} \right\}^{-1} \quad . \quad (2)$$

In Ref. [4] we have studied an intriguing property of the propagator (2), namely that it vanishes for  $E \rightarrow E_{NL}$ . Here, we will concentrate on the fact that the scattering amplitude (1) is independent of the way quark confinement is introduced, as only the confinement spectrum  $E_{NL}$  appears in expressions (1) and (2). Hence, whatever one's preferred mechanism for confinement, the scattering amplitude only depends on the resulting spectrum. As a consequence, one merely has to deduce from experiment a suitable set of values of  $E_{NL}$  and then try to guess the corresponding dynamics for confinement.

However, in the recent past we have found that analysing experimental data is far from trivial. For instance, the expressions (1) and (2) may also lead to dynamically generated resonances, like the light scalar-meson nonet [5], or the  $D_{s0}(2317)$  [6], which

do not stem directly from the confinement spectrum. Furthermore, the mountain-shaped threshold enhancements in plots of events versus invariant mass for particle production can easily be mistaken for resonances [7]. Also, to properly analyse certain hadronic decay modes, one has to turn the resulting data upside down, so as to find the true quarkonium resonances and threshold enhancements [8], instead of erroneously classifying the leftovers as unexpected new resonances [9]. Moreover, in studying resonances from production processes, one has no control over their quantum numbers [10].

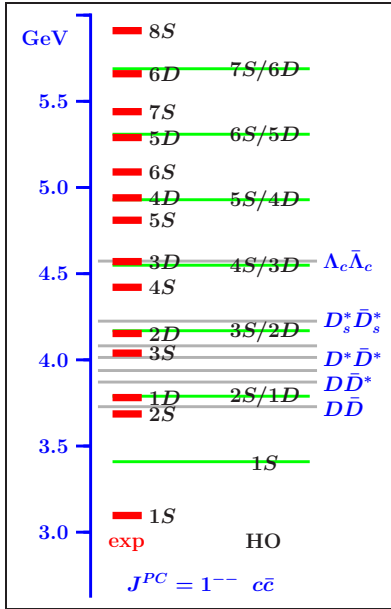
Our simple formulas (1) and (2) for meson-meson scattering are certainly not good enough for a detailed description of production processes, but must be adapted in order to account for, at least, the threshold enhancements [11]. However, the precise dynamics of production processes is still far from being fully understood. Nevertheless, for the low-lying part of the spectra we may deduce some properties without too much dependence on a specific confinement spectrum. We found that meson loops, which are properly accounted for in expressions (1) and (2), have most influence on the mass shifts of the ground states. Consequently, upon deducing a confinement spectrum from the lowest-lying states, one is urged to seriously consider the meson loops [2].

Threshold enhancements are more conspicuous for sharp thresholds, i.e., when the involved particles have small widths, rather than for diffuse thresholds, concerning decay products that have considerable widths themselves [12]. The latter phenomenon tends to happen higher up in the spectrum. There, we may expect a smoothened-out pattern of overlapping broad threshold enhancements. Therefore, higher radial excitations of quarkonium resonances can more easily be disentangled from other enhancements. The disadvantage is that any confinement mechanism predicts, for higher excitations, abundantly many states of the  $q\bar{q}$  propagator, with a variety of different quantum numbers.

Now, in order to avoid a large number of partly overlapping resonances, one best studies resonances obtained in electron-positron annihilation, which process is dominated by vector quarkonia. But this is not the full solution for cleaning up the data, since in the light-quark sector one has nonstrange and strange  $q\bar{q}$  combinations with comparable spectra, which will come out on top of each other, besides possibly significant mixing of isoscalar  $n\bar{n}$  ( $n = u, d$ ) and  $s\bar{s}$  states. Moreover, decay channels involving kaons are common to both  $n\bar{n}$  and  $s\bar{s}$  resonances. Actually, the only system with a sufficient number of established states to find evidence (see Table 3 of Ref. [13]) for a regular level splitting of about 380 MeV is given by the radially excited  $f_2$  mesons. A way out is to study a well-isolated system, with just one set of quantum numbers, like vector  $c\bar{c}$  states, which can be produced in  $e^+e^-$  annihilation. Once the spectrum of vector  $c\bar{c}$  is well established, one can with some confidence apply its properties to other spectra. Unfortunately, the well-established  $J^{PC} = 1^{--}$   $c\bar{c}$  spectrum anno 2010 still consists of  $J/\psi$ ,  $\psi(2S, 3S, 4S)$ , and  $\psi(1D, 2D)$  only.

In the following, we will concentrate on a specific choice for confinement, namely the harmonic oscillator (HO), though not so much the corresponding potential or geometry (anti-De-Sitter [14]), but just the HO spectrum that follows from these approaches. For vector  $c\bar{c}$  systems, one has a single  $^3S_1$  ground state, and radial excitations, which can be either  $^3S_1$  or  $^3D_1$ . In the HO spectrum,  $^3S_1$  states with radial quantum number  $n$  and  $^3D_1$  states with  $n-1$  are degenerate. However, due to the interaction generated by the meson loops, the poles associated with the resonances repel each other in such a way that one is subject to a small mass shift, whereas the other shifts considerably

which is dominantly  $^3S_1$ , becomes of the order of 150–200 MeV, whereas the higher pole, mostly  $^3D_1$ , acquires a central resonance position that is only a few to at most about 50 MeV away from the HO spectrum [2]. In Ref. [15] we found evidence, in data obtained by the BABAR Collaboration [16], for further charmonium states, viz.  $\psi(5S, 6S, 7S, 8S)$  and  $\psi(3D, 4D, 5D, 6D)$ , which confirm the above observation. In Fig. 1 we display the resulting spectrum for vector charmonium.



**Fig. 1:** The higher charmonium vector states (**exp**) as extracted by us from data: (i) the  $\psi(3D)$  [8], in BABAR data [9] on  $e^+e^- \rightarrow J/\psi\pi^+\pi^-$ ; (ii) the  $\psi(5S)$  and  $\psi(4D)$  [17], in data obtained by the Belle Collaboration on  $e^+e^- \rightarrow \Lambda_c^+\Lambda_c^-$  [18],  $D^0D^{*-}\pi^+$  [19], and  $D^0D^{*-}\pi^+$  [20], as well as in the missing signal of Ref. [9], and in further BABAR data on  $D^*\bar{D}^*$  [16]; (iii) the  $\psi(5S)$ ,  $\psi(4D)$ ,  $\psi(6S)$ , and  $\psi(5D)$  [21], in the data of Ref. [18]; (iv) the  $\psi(3D)$ ,  $\psi(5S)$ ,  $\psi(4D)$ ,  $\psi(6S)$ , and  $\psi(5D)$  [22], in new, preliminary BABAR data [23] on  $e^+e^- \rightarrow J/\psi\pi^+\pi^-$ ; (v) the  $\psi(7S)$ ,  $\psi(6D)$ , and  $\psi(8S)$  [15], in data from BABAR on  $D^*\bar{D}^*$  [16]. We also indicate the level scheme as predicted by pure HO confinement (HO) (**—**). Meson and baryon loops shift the  $D$  states a few MeV down/up, whereas the  $S$  states shift 100–200 MeV downwards. For completeness, we also indicate the levels of the sharp, low-lying meson-meson and baryon-baryon thresholds (**—**) of the channels  $D\bar{D}$ ,  $D\bar{D}^*$ ,  $D_s\bar{D}_s$ ,  $D^*\bar{D}^*$ ,  $D_s\bar{D}_s^*$ ,  $D_s^*\bar{D}_s^*$ , and  $\Lambda_c^+\Lambda_c^-$ .

We can conclude from Fig. 1 that our guess of an HO spectrum for vector  $c\bar{c}$  states, with a radial level spacing of 380 MeV [24], seems to work well in view of the data. Furthermore, we may observe the advantage of studying the  $c\bar{c}$  vector spectrum from above, where the pattern of dominantly  $S$  and dominantly  $D$  states becomes rather regular.

**Summarizing**, we have shown that the  $c\bar{c}$  confinement spectrum, which underlies scattering and production of multi-meson systems containing open-charm pairs, can best be observed by starting from higher radial excitations of vector charmonium in electron-positron annihilation. Moreover, we have shown that a constant radial level splitting of about 380 MeV is consistent with light and heavy meson spectra.

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